MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”
Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 14

Generalized Displacements in Two-Supported Beams in Plane Bending

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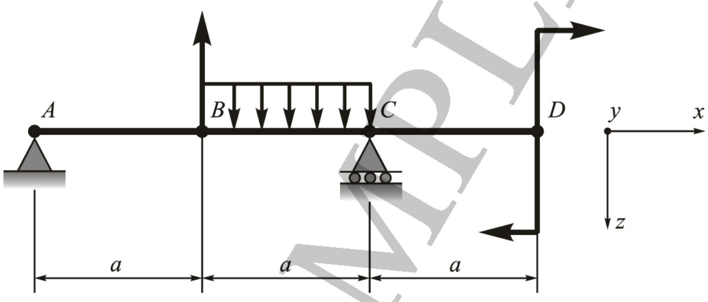

Given: $q = 10$ kN/m; $P = 20$ kN; $M = 10$ kNm; $E = 2 \times 10^{11}$ Pa; $[\sigma] = 160$ MPa; $a = 2$ m.
Goal:
1) calculate dimensions of the cross-section choosing the one of following: a) diameter of the round solid; b) dimensions of the rectangle (*h*/*b*=2); c) I-beam number;
2) calculate vertical displacement and the slope in the following points:

$\theta_A - ?$ $\theta_B - ?$ $\theta_C - ?$ $\theta_D - ?$
$z_B - ?$ $z_D - ?$

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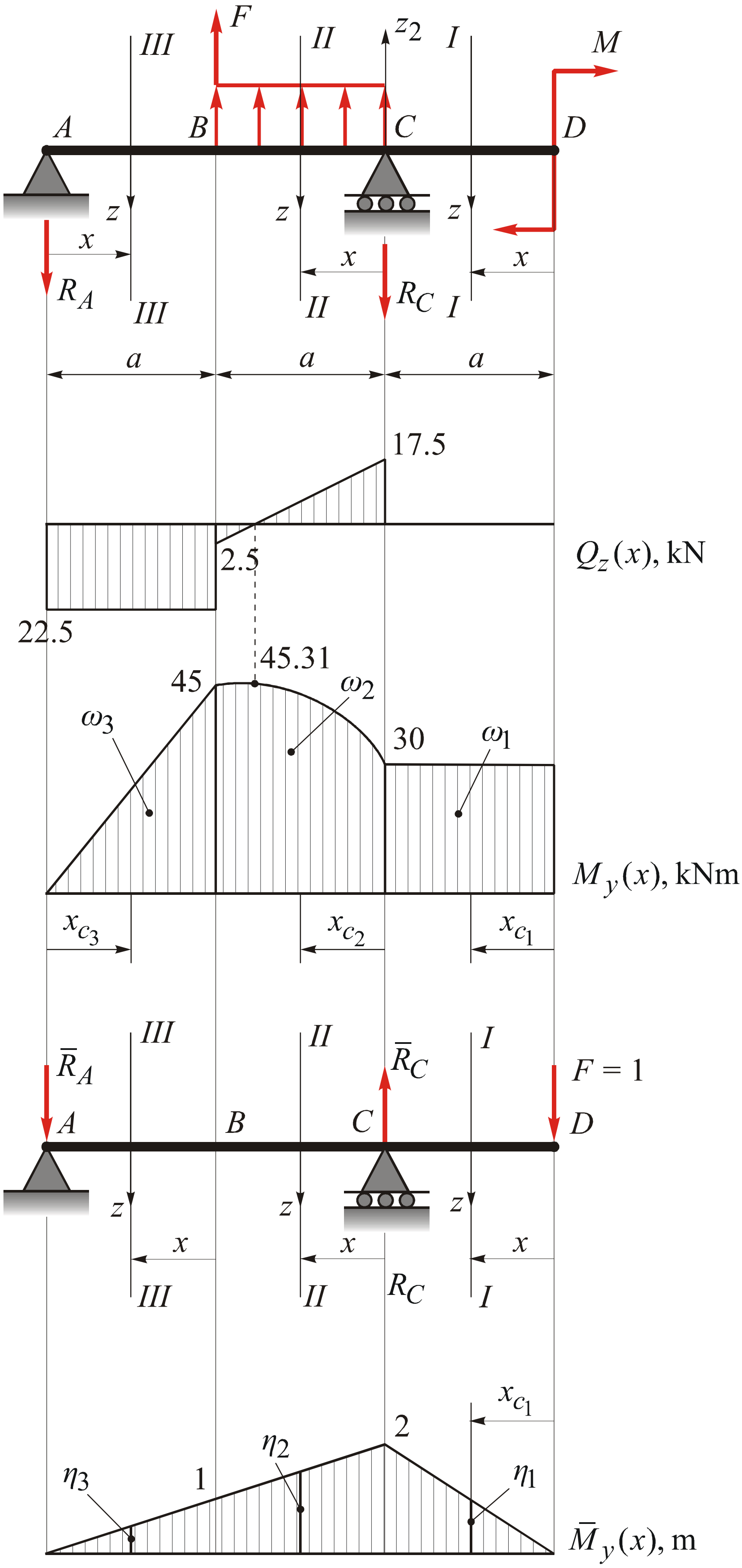

Fig. 1

Given: $q=10$ кН/m, $F=20$ kN, M=30kNm, $a=2$ m, $E=2\times10^{11}$ Pa, $[s]=100$ MPa.

It is necessary:

1) To determine the diameter of the circular cross-section;

2) To calculate the vertical displacement (deflection) of the D section.

Note: to solve the problem by Mohr's energy method and Vereshchagin's graphical method

In a plane bending, the Mohr's integral is:

$$d = \sum_{i=1}^{n} \int_{l_i} \frac{M_{yF}(x)\bar{M}_y(x)}{EI_y} dx,$$

where $M_{yF}(x)$ – bending moment in an arbitrary section of the force system (F), $\bar{M}_y(x)$ – bending moment in an arbitrary section of the unit system (1);

EI_y – flexural rigidity.

Vereshchagin's formula is:

$$d = \sum_{i=1}^{n} \frac{w_i h_i}{EI_y},$$ where w_i – area of the bending moment graph part for the force system; h_i – ordinate of unit diagram under centroid of force system bending moment graph.

Solution

1. Let us write the equations of internal forces in arbitrary sections of given (force) system. Before this, let us find support reactions R_A and R_C from equilibrium conditions. We will originally direct the reactions downwards. Then

$$\sum M_A = 0 = 2R_C a + M - Fa - qa\left(\frac{a}{2} + a\right)$$

$$R_C = \frac{1}{2a}\left(Fa + qa\left(\frac{a}{2} + a\right) - M\right) = \frac{1}{4}(20\times 2 + 20\times 3 - 30) = \frac{1}{4}(70) = +17{,}5\,\text{kN}.$$

$$\sum M_C = 0 = M + Fa + qa\left(\frac{a}{2}\right) - 2R_A a,$$

$$R_A = \frac{1}{2a}(M + Fa + qa(\frac{a}{2})) = \frac{1}{4}(30 + 40 + 20) = +22{,}5\,\text{kN}.$$

Checking: $\sum F_z = 0 = R_A + R_C - qa - F = 17{,}5 + 22{,}5 - 20 - 20 = 0$.

Let us divide the force system into parts as shown in Fig. 1 and write equations of shear forces and bending moments for every part:

$I - I \quad 0 < x < a$

$$Q_{zF}^{I}(x) = 0$$

$$M_{yF}^{I}(x) = -M = -30\,\text{kNm}$$

$II - II \quad 0 < x < a$

$$Q_{zF}^{II}(x) = R_C - qx\Big|_{x=0} = 17{,}5\Big|_{x=2} = -2{,}5\,\text{кН}$$

$$M_{yF}{}^{II}(x) = -M - R_C x + \frac{qx^2}{2}\Bigg|_{x=0} = -30\Big|_{x=2} = -45{,}31\,\text{кН}$$

Since the graph of shear force changes its sign within the limits of second portion, bending moment function will have an extremal value:

$$Q_{zF}^{II}(x_e) = R_C - qx_e = 0 \rightarrow x_e = \frac{R_C}{q} = \frac{17{,}5}{10} = 1{,}75\,\text{m}.$$

$$M_{yF}^{II}(x_e) = M_{y\max} = -M - R_c x_e - q(x_e)^2/2 =$$

$$= -30 - 17.5\times 1.75 - 10\times(1.75)^2/2 = -45.3\,\text{kNm}.$$

$III - III \quad 0 < x < 2$

$$Q_{zF}^{III}(x) = -R_A = -22{,}5\,\text{kN}$$

$$M_{yF}^{III}(x) = -R_A x\Big|_{x=0} = 0\Big|_{x=2} = -45\,\text{кНм}$$

2. Design the graphs of shear forces and bending moments for the portions of the force (F) system, also calculate the areas of bending moment graphs w_i and coordinates of their centroids x_{c_i} (see Fig. 1).

$$w_1 = \int_0^a M_{yF}^{I}(x)\,dx = \int_0^a (-30)dx = -60 \text{ kNm}^2, \quad x_{c_1} = 1 \text{ m},$$

$$w_2 = \int_0^a M_{yF}^{II}(x)\,dx = \int_0^a \left(-M - R_c x + \frac{ax^2}{2} \right) dx = -\int_0^a M dx - R_c \int_0^a x dx +$$

$$+\frac{a}{2}\int_0^a x^2 dx = -60 - 17.5 \times \frac{2^2}{2} + 5\frac{2^3}{3} = -60 - 35 + \frac{40}{3} = -81.67 \text{ kNm}^2.$$

We will define the centroidal coordinate of this area by formula $x_{c_2} = S_{z_2}/w_2$, where S_{z_2} – static moment of area w_2, calculated relatively to z_2 axis, passing through C point as the origin of second portion.

$$S_{z_2} = \int_0^2 M_y^{II}(x)\,x dx = \int_0^a \left(-M - R_c x + \frac{ax^2}{2} \right) x dx =$$

$$= \int_0^a Mx dx - \int_0^a R_c x^2 dx + \int_0^2 \frac{qx^3}{2} dx = -M\frac{x^2}{2}\Bigg|_0^a - R_c \frac{x^3}{3}\Bigg|_0^a + \frac{qx^4}{2 \times 4}\Bigg|_0^a = -30\frac{4}{2} - 17.5\frac{8}{3} +$$

$$+\frac{5 \times 16}{4} = -60 - 46.67 + 20 = -86.67 \text{ kNm}^3.$$

Then $x_{c_2} = (-86.67)/(-81.67) = 1.06$ m.

Note. Calculating these areas and coordinates is necessary for further calculation of generalized displacements by Vereshchagin's method.

Conclusion: $II - II$ portion is critical and $\left| M_{y_{\max}} \right| = 45.3$ kNm.

3. Determine the diameter of the circular cross section from the condition of strength:

$$s_{\max} = \frac{\left| M_{y_{\max}} \right|}{W_y} \le [s]; \quad W_y = \frac{\left| M_{y_{\max}} \right|}{[s]} = \frac{45.31 \times 10^3}{160 \times 10^6} = 283 \times 10^{-6} \text{ m}^3.$$

From the other side, $W_y = \dfrac{p D^3}{32}$, then

$$D \ge \sqrt[3]{\frac{32 W_y}{p}} = \sqrt[3]{\frac{32 \times 283 \times 10^{-6}}{3.14}} = 14.23 \text{ cm}.$$

Calculate the axial moment of inertia of the cross sections found to determine in future cross-sectional flexural rigidity IE:

$$I_y = \frac{p D^4}{64} = \frac{3.14(0.1423)^4}{64} = 2011.7 \times 10^{-8}\ \text{m}^4.$$

4. Determine the deflection of D section by the Mohr's energy method. Given beam we will consider as force system (F) and also should design corresponding unit system (1). It is designed on a Fig. 2 by applying unit dimensionless force $\overline{F} = 1$ in D point in vertical direction (downwards, for example). It is necessary also to draw its graph of unit bending moments. Preliminary, let us calculate the reactions in supports of unit system $\overline{R_A}$ и $\overline{R_C}$ from equations of static equilibrium.

$$\sum M_A = 0 = \overline{F} \times 3a - \overline{R_C} \times 2a \rightarrow \overline{R_C} = 1.5 \text{ (dimensionless)}$$
$$\sum M_C = 0 = \overline{F} \times a - \overline{R_A} \times 2a \rightarrow \overline{R_A} = 0.5 \text{ (dimensionless)}$$

Checking: $\sum F_z = 0 = \overline{R_C} - \overline{F} - \overline{R_A} = 1.5 - 1 - 0.5 = 0$.

By identical dividing both systems onto portions, let us write equations of bending moments in the most simple shape to be suitable for Mohr's integral substituting:

$I - I \quad 0 < x < 2$

$$M_{yF}^{I}(x) = -M = -30, \text{ kNm}$$
$$\overline{M}_y^{I}(x) = -1 \times x, \text{ m}$$

$II - II \quad 0 < x < 2$

$$M_{yF}^{II}(x) = -M - R_C x + \frac{qx^2}{2} = -30 - 17.5x + 5x^2, \text{ kNm}$$
$$\overline{M}_y^{II}(x) = -\overline{F}(2 + x) + \overline{R_C} x = -2 - x + 1.5x = -2 + \frac{1}{2}x, \text{ m}$$

$III - III \quad 0 < x < 2$

$$M_{yF}^{III}(x) = -R_A x = -22.5x, \text{ kNm}$$
$$\overline{M}_y^{III}(x) = -\overline{R_A} x = -\frac{1}{2}x, \text{ m}.$$

Substituting in Mohr's integral and integrating, we obtain:

$$z_D = \frac{1}{EI_y}\left[\int_0^2 (-30)(-x)\,dx + \int_0^2 \left(-30 + 17.5x + 5x^2\right)\left(-2 + \frac{x}{2}\right)dx + \int_0^2 (-22.5x)(-\frac{x}{2})dx\right] =$$

$$= \frac{1}{EI_y}\left[30 \times 2^2 + 60 \times 2 + 35 \times \frac{2^2}{2} - 10 \times \frac{2^3}{3} - 15 \times \frac{2^2}{2} - \frac{17.5}{2} \times \frac{2^3}{3} + \frac{5}{2} \times \frac{2^4}{4} + \frac{22.5}{2} \times \frac{2^3}{3}\right] =$$

$$= +\frac{210}{EI_y}, \frac{\text{kNm}^3}{EI_y}.$$

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Substituting parameters of flexural rigidity, we obtain

$$z_D = +\frac{210\times10^3}{2\times10^{11}\times2011.7\times10^{-8}} = +52.20\times10^{-3} \text{ m=+52.2mm.}$$

Note, that (+) sign in the solution means, that real direction of point *D* deflection coincides with the initially selected, i.e. *D* point moves downwards.

To determine the desired displacement by Vereshchagin's method, we should define on the unit bending moment graph $\bar{M}_y(x)$ its three ordinates h_1, h_2, h_3, in the cross-sections with coordinates x_{c_1}, x_{c_2}, x_{c_3}, i.e. under $M_{yF}(x)$ graphs centroids. First and third of them, i.e. x_{c_1} and x_{c_3} are easy to define: since $x_{c_1} = 1\,\text{m}$, we have $h_1 = \frac{1}{2}\times2 = 1\,\text{m}$ (see. Fig. 1). It also evident that for triangle with w_3 area $h_3 = \frac{2}{3}\times2 = 1.33$ m.

To find h_2 ordinate the w_2 centroid should be used, i.e. $x_{c_2} = 1.06$ m. Also, similarity of triangles arising from Fig. 2 is used to find h_2 ordinate:

$\frac{2}{4} = \frac{h_2}{4-1.06}$, and $h_2 = \frac{1}{2}(4-1.06) = 1.47\,\text{m}$.

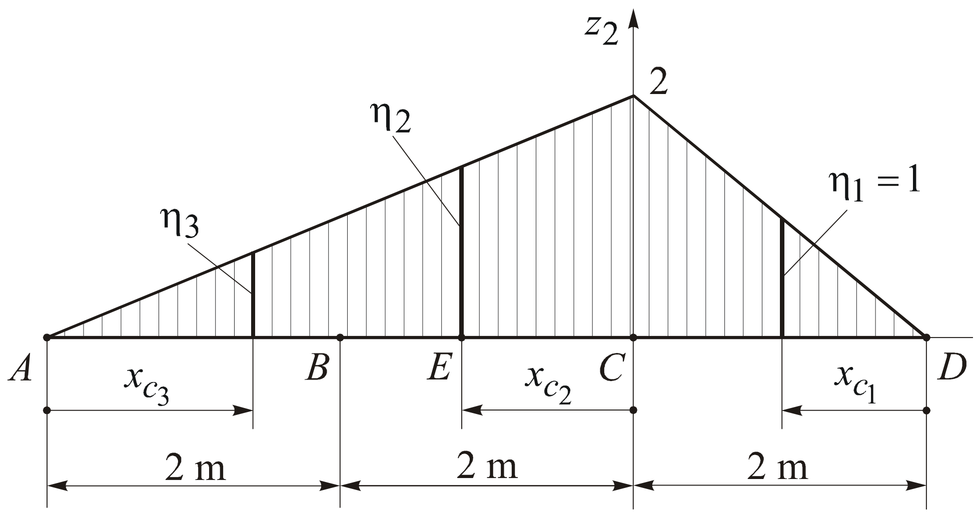

Fig. 2

Thus, the desired deflection

$$z_D = \frac{1}{EI_y}\left[\left(w_1h_1 + w_2h_2 + w_3h_3\right)\right] =$$

$$= \frac{1}{EI_y}\left[(-60)(-1) + (-81,67)(-1,47) + (-45)\left(-\frac{2}{3}\times1\right)\right] = +\frac{210,2}{EI_y},\ \frac{\text{kNm}^3}{EI_y}.$$

Substituting the parameters of flexural rigidity EI_y, we obtain

$$z_D = +\frac{210.2\times10^3}{2\times10^{11}\times2011.7\times10^{-8}} = +52.24\times10^{-3} \text{ m = +52.24 mm.}$$

Conclusion. This result coincides with Mohr's method solution.