MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National aerospace university “Kharkiv Aviation Institute”

Department of aircraft strength

Course
Mechanics of materials and structures

HOME PROBLEM 1

# Geometrical Properties of Composite Plane Area

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**Department of aircraft strength**

**Subject:** mechanics of materials
**Document:** home problem
**Topic:** geometrical properties of plane area

**Full name of the student, group**

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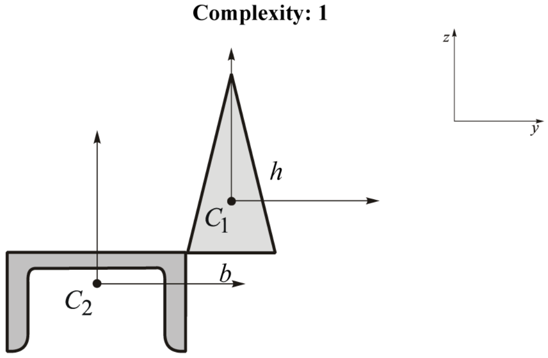
**Variant:** 1 **Complexity:** 1




**Given:** $h = 3$ cm, $b = 2$ cm, channel № 5.

**Goal:** 1) determine the coordinates of cross-sectional centroid; 2) determine the position of central principal axes of inertia and principal central moments of inertia

**Full name of the lecturer** **signature**

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**Mark:**

## Solution

1. Use the following numerical data (see Table) and draw the section in scale (Fig. 1).

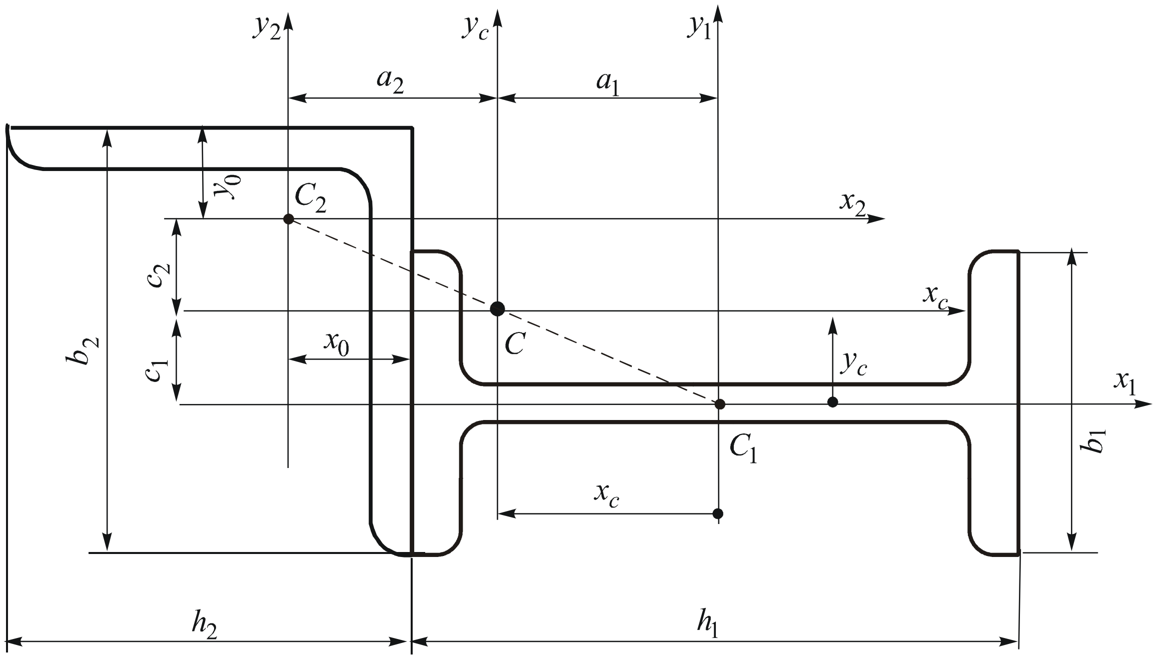



Fig. 1

Table

| Parts of the composite area | Geometrical properties | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $h_i$, m | $b_i$, m | $A_i$, m$^2$ | $I_{x_i}$, m$^4$ | $I_{y_i}$, m$^4$ | $I_{x_i y_i}$, m$^4$ | $I_{\max_i}$, m$^4$ | $I_{\min_i}$, m$^4$ | $y_0$, m |
| 1 – $\vdash\dashv$ GOST 8239-72 | 0.2 | 0.1 | $26.8\times10^{-4}$ | $115\times10^{-8}$ | $1840\times10^{-8}$ | 0 | $1840\times10^{-8}$ | $115\times10^{-8}$ | – |
| 2 – $\neg$ GOST 8509-72 | 0.16 | 0.16 | $31.4\times10^{-4}$ | $774\times10^{-8}$ | $774\times10^{-8}$ | – | $1229\times10^{-8}$ | $319\times10^{-8}$ | $4.3\times10^{-2}$ |

The coordinates of two $C_1$ and $C_2$ centroids for the parts are known from assortments ($x_0 = y_0 = 4.3\times10^{-2}$ m).

2. Calculation of the centroidal coordinates for composite area.

Axes $x_1, y_1$ are selected as reference axes in this study (see Fig. 1).

The following formulae are used

$$x_c = S_{y_1} / A, \quad y_c = S_{x_1} / A, \quad \text{where}$$

$$S_{y_1} = S_{y_1}^{\vdash\dashv} + S_{y_1}^{\neg}; \quad S_{x_1} = S_{x_1}^{\vdash\dashv} + S_{x_1}^{\neg}.$$

$$A = A^{\vdash\dashv} + A^{\neg} = 26.8\times10^{-4} + 31.4\times10^{-4} = 58.2\times10^{-4}\ \text{m}^2.$$

$S_{x_1}^{\vdash\dashv}$ and $S_{y_1}^{\vdash\dashv}$ are zero due to central character of $x_1, y_1$ axes for I-beam.

$$S_{x_1}^{\neg} = A^{\neg}(+(b_2 - \frac{b_1}{2} - y_0)) = 31.4\times10^{-4}(+(0.16 - 0.05 - 0.043)) = 31.4\times10^{-4}\times0.067 =$$

$$= +2.10\times10^{-4}\ \text{m}^3.$$

$$S_{y_1}^{\neg} = A^{\neg}(-(\frac{h_1}{2} + x_0)) = 31.4\times10^{-4}(-(0.1 + 0.043)) = -4.49\times10^{-4}\ \text{m}^3.$$

$$S_{y_1} = 0 - 4.49\times10^{-4} = -4.49\times10^{-4}\ \text{m}^3.$$

$$S_{x_1} = 0 + 2.10\times10^{-4} = +2.10\times10^{-4}\ \text{m}^3.$$

$$x_c = -4.49\times10^{-4} / 58.2\times10^{-4} = -0.077\ \text{m} = -7.715\ \text{cm}.$$

$$y_c = +2.10\times10^{-4} / 58.2\times10^{-4} = +0.03615\ \text{m} = +3.615\ \text{cm}.$$

Results: the coordinates of the $C$ centroid are equal to:

$$x_c = -7.715\times10^{-2}\ \text{m},$$

$$y_c = 3.615\times10^{-2}\ \text{m}.$$

They are shown on Fig. 1.

3. Calculation of central moments of inertia relative to central $x_c, y_c$ axes.

Let us denote the $x_c$, $y_c$ axes as the centroidal axes of the composite area. The moments and product of inertia with respect to these axes can be obtained using the parallel-axis theorems. The results of such calculations are as follows.

$$I_{x_c} = I_{x_c}^{\vdash\dashv} + I_{x_c}^{\neg},$$

$$I_{x_c}^{\vdash\dashv} = I_{x_1}^{\vdash\dashv} + c_1^2 A_1 = 115\times10^{-8} + 3.615^2\times26.8\times10^{-8} = 465.23\times10^{-8}\ \text{m}^4,$$

$$I_{x_c}^{\neg} = I_{x_2}^{\neg} + c_2^2 A_2 = 774\times10^{-8} + 3.085^2\times31.4\times10^{-8} = 1072.8\times10^{-8}\ \text{m}^4,$$

$$I_{x_c} = (465.23 + 1072.8)10^{-8} = 1538\times10^{-8}\ \text{m}^4.$$

$$I_{y_c} = I_{y_c}^{\vdash\dashv} + I_{y_c}^{\neg},$$

$$I_{y_c}^{\vdash\dashv} = I_{y_1}^{\vdash\dashv} + a_1^2 A_1 = 1840\times10^{-8} + 7.715^2\times26.8\times10^{-8} = 3435.2\times10^{-8}\ \text{m}^4,$$

$$I_{y_c}^{\neg} = I_{y_2}^{\neg} + a_2^2 A_2 = 774\times10^{-8} + 6.585^2\times31.4\times10^{-8} = 2135.6\times10^{-8}\ \text{m}^4,$$

$$I_{y_c} = (3435.2 + 2135.6)10^{-8} = 5570.8\times10^{-8}\ \text{m}^4.$$

4. Calculation of the product of inertia relative to $x_c, y_c$ axes.

$$I_{x_c y_c} = I_{x_c y_c}^{\vdash\dashv} + I_{x_c y_c}^{\neg},$$

For the first part of the section:

$$I_{x_c y_c}^{\vdash\dashv} = I_{x_1 y_1}^{\vdash\dashv} + a_1 c_1 A_1 = 0 + 7.715(-3.615)\times10^{-4}\times26.8\times10^{-4} = -747.4\times10^{-8}\ \text{m}^4.$$

For second part the similar approach is used:

$$I_{x_c y_c}^{\neg} = I_{x_2 y_2}^{\neg} + a_2 c_2 A_2.$$

The value of $I_{x_2 y_2}^{\neg}$ should be determined beforehand using transformation equations for product of inertia and taking into account that in rotation of axes the sum of axial moments of inertia is unchanged, i.e. $I_{x_2} + I_{y_2} = I_{\max} + I_{\min}$. The axes rotating procedure is shown in Fig. 2. The $x_3$, $y_3$ axes are selected as reference axes in this rotation to $x_2$, $y_2$ axes.

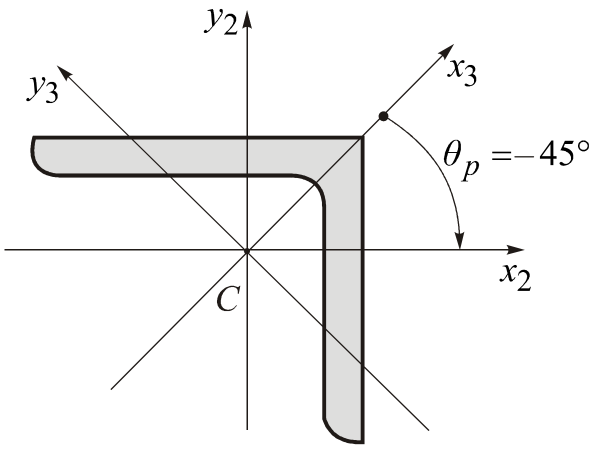


Fig. 2

Due to cross-section symmetry relative to $x_3$ axis, the angle of rotation is $q_p = -45°$ (clockwise rotation).

In our case, general view of transformation equation for product of inertia

$$I_{x_1 y_1} = \frac{I_x - I_y}{2}\sin 2q + I_{xy}\cos 2q$$

will be rewritten as

$$I_{x_2 y_2} = \frac{I_{x_3} - I_{y_3}}{2}\sin 2q_p + I_{x_3 y_3}\cos 2q_p.$$

After substituting,

$$I_{x_2 y_2} = \frac{1229\times10^{-8} - 319\times10^{-8}}{2}\sin(-90°) + 0\cos(-90°) = -455\times10^{-8}\ \text{m}^4.$$

In our designations, this product will be denoted as $I_{x_2 y_2}^{\neg} = -455\times10^{-8}\ \text{m}^4$.

Consequently,

$$I^{\neg}_{x_c y_c} = -455\times10^{-8} + (-6.585)(3.085)\times31.4\times10^{-8} = -1092.9\times10^{-8}\ \text{m}^4 .$$

Total result after substitutions is

$$I_{x_c y_c} = (-747.4 - 1092.9)\times10^{-8} = -1840.3\times10^{-8}\ \text{m}^4 .$$

5. Rotating central $x_c, y_c$ axes to central principal position at $q_p$ angle.

Substituting the values of central moments and product of inertia into the equation for the angle $q_p$, we get

$$\text{tg}2q_p = \frac{2I_{x_c y_c}}{I_{y_c} - I_{x_c}} = \frac{2\times(-1840.3)}{5570.8 - 1538} = -0.9127 \Rightarrow 2q_p = -42^{0}24' \Rightarrow q_p = -21^{0}12'.$$

**Note, that this angle is clockwise due to used sign convention. It is shown in resultant picture shown below (Fig. 3).**

It is important to note that in any rotation of axes to principal position larger of two axial moments of inertia ($I_{y_c} = 5570.8\,\text{cm}^4$) becomes the largest (maximum) and smaller one ($I_{y_c} = 1538\,\text{cm}^4$) becomes the minimum in value.

6. Calculation of principal central moments of inertia for composite area.

The principal moments of inertia are determined using the formula

$$I_{UV} = I_{\substack{\max\\ \min}} = \frac{I_{x_c} + I_{y_c}}{2} \pm \sqrt{\left(\frac{I_{x_c} - I_{y_c}}{2}\right)^2 + I^2_{x_c y_c}} = \left(3554.4 \pm 2293.2\right)\times10^{-8}\ \text{m}^4 ,$$

$$I_U = I_{\max} = 5847.6\times10^{-8}\ \text{m}^4, \quad I_V = I_{\min} = 1261.2\times10^{-8}\ \text{m}^4 .$$

**Note, both values must be positive!**

7. Checking the results:

(a) Checking the correspondence: $I_{\max} > I_{y_c} > I_{x_c} > I_{\min}$ (in the case $I_{y_c} > I_{x_c}$) or
$I_{\max} > I_{x_c} > I_{y_c} > I_{\min}$ (in the case $I_{x_c} > I_{y_c}$ ).

In our case, $5847.6\times10^{-8} > 5570.8\times10^{-8} > 1538\times10^{-8} > 1261.2\times10^{-8}$.

(b) Checking the constancy of the sum of axial moments of inertia in rotating the axes:

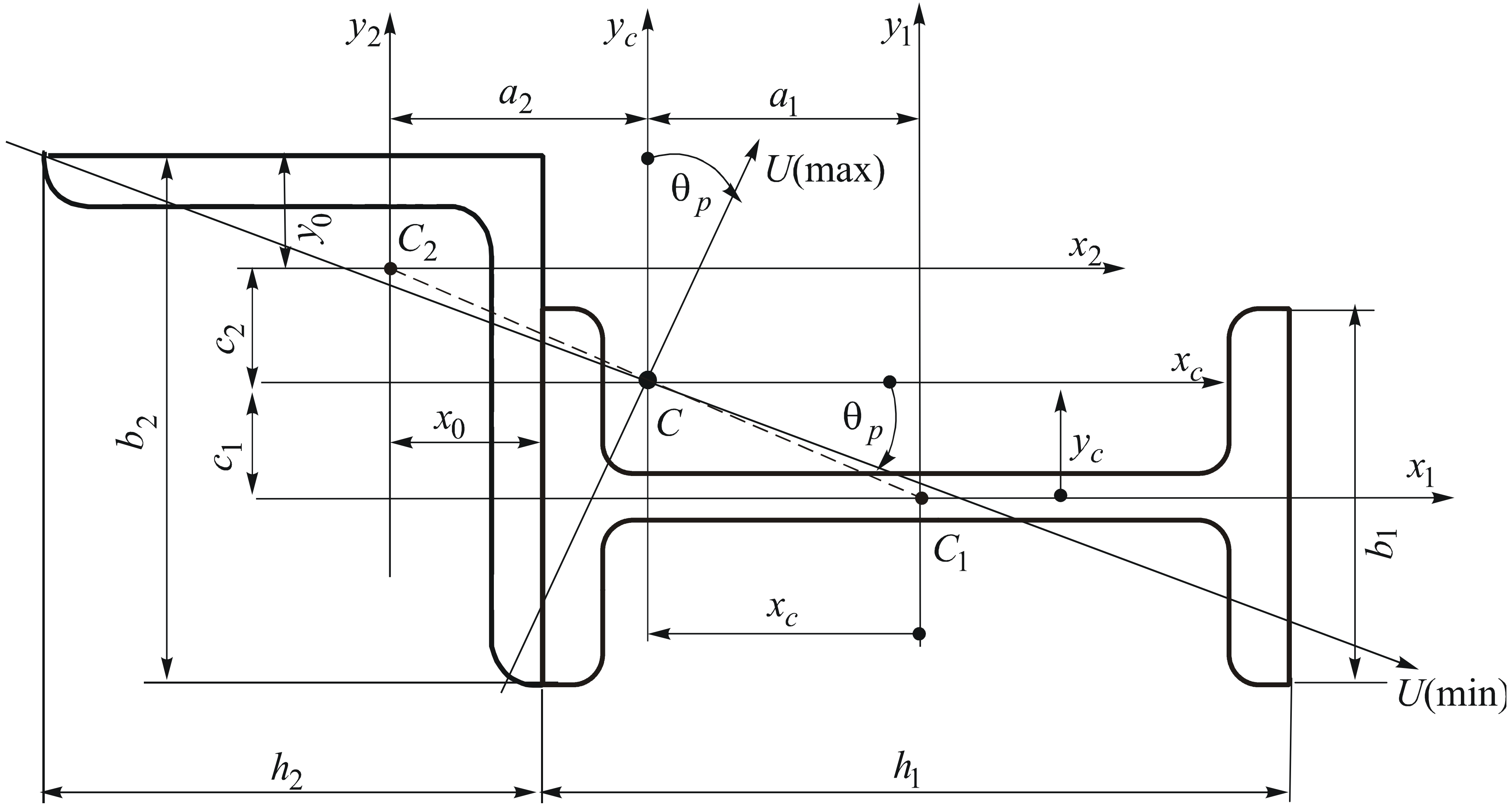
$$I_{\max} + I_{\min} = I_{x_c} + I_{y_c} , \rightarrow\ 5847.6\times10^{-8} + 1261.2\times10^{-8} = 5570.8\times10^{-8} + 1538\times10^{-8} ,$$

$$\left(7108.8\times10^{-8} = 7108.8\times10^{-8}\right).$$

(c) Calculating the evidently zero central principal product of inertia of the section:

$$I_{UV} = I_{x_c y_c}\cos 2q_p + \frac{I_{y_c} - I_{z_c}}{2}\sin 2q_p =$$

$$= \left[(-1804.3)\times0.7384 + \frac{1538 - 5570.8}{2}\times\left(-0.6743\right)\right]\times10^{-8} = \left(-1358.9 + 1359\right)\times10^{-8}\ \text{m}^4 \cong 0.$$



y2
yc
y1
a2
a1
U(max)
θp
y0
C2
x2
c2
xc
C
b2
x0
c1
θp
yc
x1
C1
b1
xc
U(min)
h2
h1


Fig. 3